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ACOUSTIC IMPEDANCE AND ITS MEASUREMENT.

By A. E. Kennelly and K. Kurokawa.

(Continued from page 3 of cover.)

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It is the object of this paper to describe some measurements of acoustic impedance and to offer a technique for such measurements in the laboratory.

Definitions of Acoustic Impedance and Acoustic Impedance Density: If a small thin rigid disk, or circular diaphragm AB, Figure 1, fitting without edge friction, in a smooth cylindrical tube TT, T″T′, of S sq. cm. internal area, be actuated, through the central rod, by a simple alternating mechanical force, or vibromotive force (vmf.) F root-mean-square dynes, directed along the axis of the tube; then the effect of the force, neglecting the friction of the air on the walls of the tube, will be to set up an alternating velocity $\dot{x}$ cm. per second, in the

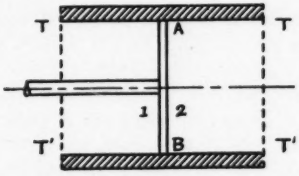

Figure 1. Diagram of Longitudinal Section of an Acoustic Tube with Vibrating Disk.

air or other fluid contained in the tube. The maximum cyclic amplitude of vibration $\pm x_m$ cm. about the quiescent position, is supposed to be very small. The fluid filling the tube on each side of the disk may be compressible, as in the ordinary case of an air tube. The maximum cyclic velocity of the fluid $\dot{x}_m$ in contact with the disk, must be the same as the maximum cyclic velocity of the disk itself. In the case of a rigid disk, this velocity would be the same at all parts of its surface. The magnitude and phase of this velocity for a given vmf. will depend upon the impedance to vibration of the column of fluid on each surface. This impedance to vibration velocity, of the fluid in a tube at a driving section, may be called the *acoustic impedance* of the fluid, at each of the two surfaces of the disk.

Acoustic impedance is the planevector sum of *acoustic resistance* and *acoustic reactance*. Acoustic resistance absorbs and dissipates the energy of acoustic vibration. Acoustic reactance cyclically stores and releases, without dissipation, the energy of acoustic vibration. If we denote the acoustic impedance of the fluid on sides 1 and 2 of the disk by $\mathbf{z}_1$ and $\mathbf{z}_2$ respectively; then the vector root-mean-square velocity of vibration at the disk surface will be:

$$\dot{x} = \frac{\mathrm{F}}{\mathbf{z}_1 + \mathbf{z}_2} = \frac{\mathrm{F}}{\mathbf{z}} \qquad \text{rms. kines,}^1 \text{ or cm. per sec.} \angle \quad (1)$$

where $\mathbf{z}$ is the total acoustic impedance to velocity of the fluid in the tube, at the disk,

$$\text{or} \qquad \mathbf{z} = \frac{\mathrm{F}}{\dot{x}} \qquad \begin{matrix} \text{acoustic absohms} \angle \\ \text{or dynes per kine} \angle \end{matrix} \quad (2)$$

F being taken as of standard phase.

If we employ the C. G. S. system of units throughout, the acoustic impedances $\mathbf{z}_1$ and $\mathbf{z}_2$ may be expressed in vector acoustic absohms. An absohm in acoustic resistance is therefore such a total resistance to vibration at the surface of a disk, subjected to a simple harmonic vibromotive force of 1 rms. dyne, as will cause a vibrational velocity to be produced of 1 cm. rms. per second, in phase with the vmf. An acoustic impedance is, however, a planevector quantity, having both a size and a slope, and is expressible in complex numbers. If the slope of $\mathbf{z}$ is $+\beta°$, the phase of the velocity will be retarded, by (1), through $\beta°$ behind the vmf. Acoustic impedance is presented by the air in contact with a vibrating telephone diaphragm, and measurements made of the impedance offered by the air to a telephone diaphragm have shown the need for a definition of this quantity and of its unit.

If the tube, on its two sides 1 and 2, is perfectly symmetrical with respect to the disk AB, the two impedances $\mathbf{z}_1$ and $\mathbf{z}_2$ must be equal at least in size if not in slope; but whether equal or not, their sum will be equal to a total vector impedance $\mathbf{z}$ acoustic absohms $\angle$.

The impedance $\mathbf{z}$ will manifestly increase as the area of the tube is increased. Neglecting friction at the walls of the tube, the increase will be in direct proportion to the area. If S be the area of the tube in sq. cm., then the acoustic impedance per sq. cm., or *acoustic im-*

1 The kine is a name of the C. G. S. unit of linear velocity, or cm./sec., as proposed at a B. A. meeting.

pedance density, will be $\mathfrak{z} = \frac{\mathbf{z}}{S}$ vector acoustic absohms per sq. cm.

Or, if we employ a vibromotive pressure intensity $p = \frac{F}{S}$ vector maximum cyclic dynes per sq. cm. to actuate the disk; then p being taken with standard phase:

$$\mathfrak{z} = \frac{p}{\dot{x}} \qquad \frac{\text{acoustic absohms}}{\text{sq. cm.}} \angle \qquad (3)$$

or

$$\dot{x} = \frac{p}{\mathfrak{z}} = \frac{p}{\mathfrak{z}_1 + \mathfrak{z}_2} \qquad \text{rms. cm. per sec.} \angle \qquad (4)$$

The dimensions of acoustic impedance, as derived from (2), are MT^{-1}, or mass-per unit of time (force divided by velocity). The dimensions of acoustic impedance density, as derived from (3) are $ML^{-2}T^{-1}$ or MT^{-1}/L^2.

The acoustic impedance of a fluid at the surface of a vibrating diaphragm is therefore the opposition to the development of vibrational velocity at that surface under impressed vmf. Although when so defined, acoustic impedance involves the existence of a diaphragm or mechanical surface, at which the impedance is produced; yet acoustic impedance may be conceived of as occurring at an imaginary surface, such as the cross section of an acoustic tube, and without the interposition of a diaphragm. Moreover, acoustic impedance is not confined to a tube, but may occur in a region of any shape.

We have hitherto assumed that the velocity $\dot{x}$ was the same at all parts of the vibrating disk or diaphragm. In any actual flexible diaphragm, however, such as a telephone-receiver diaphragm, the vibrational displacement x, and velocity $\dot{x}$, will be different at different distances from the center of the disk. If we take any elementary area dS of the surface of the disk, at which the velocity is $\dot{x}$ rms. kines, the power of this motion is

$$dP = \dot{x}^2\, \mathfrak{z}\, dS \qquad \text{abwatts or ergs per sec.} \angle \qquad (5)$$

where $\dot{x}$ is taken as of standard phase and zero slope. Of this power, the real component is dissipated, and the imaginary component is cyclically stored and released. The total power delivered to the disk, including both sides, will be

$$P = \int_o^S \dot{x}^2\, \mathfrak{z}\, dS = \mathfrak{z} \int_o^S \dot{x}^2\, dS \qquad \text{abwatts} \angle \qquad (6)$$

$\mathfrak{z}$ being assumed constant over the entire surface of the diaphragm. This power may be expressed as

$$P = \underline{\dot{x}}^2 \, \mathfrak{z} \, S \qquad \text{abwatts} \qquad (7)$$

where

$$\underline{\dot{x}}^2 = \frac{1}{S} \int_o^s \dot{x}^2 \, dS \qquad \text{(rms. kines)}^2 \qquad (8)$$

or the vector mean velocity square $\underline{\dot{x}}^2$ is the integrated mean value of $\dot{x}^2$ over the entire surface. Under these conditions, the acoustic impedance of the tube at the disk including both of its sides, will be

$$\mathfrak{z} = \frac{F}{\underline{\dot{x}}} \qquad \text{acoustic absohms} \angle \qquad (9)$$

The mean square ratio $\left(\frac{\underline{\dot{x}}}{\dot{x}_o}\right)^2 = \left(\frac{\underline{x}}{x_o}\right)^2$ of average to maximum central velocity or amplitude has been called the mass factor [2] of the diaphragm denoted by $\frac{m}{M}$.

Total Mechanic Impedance on a Diaphragm: When a telephone diaphragm is set in vibration under an impressed vmf., due to a simple harmonic alternating current in the coils, the maximum cyclic velocity of the diaphragm over the poles depends upon the total mechanic impedance to motion. The vector vmf. F may be taken as proportional to the exciting current I rms. absamperes flowing through the coils.[3]

$$F = A \, I \qquad \text{rms. dynes} \angle \qquad (10)$$

Here A is the vector force factor of the instrument, and ordinarily has a slope $\beta°$, of about $-30°$, so that the vmf. F has the same frequency as I, and lags in phase behind I by this angle $\beta°$. The total mechanic impedance $\mathfrak{z}'$ of the diaphragm limits the max. cyclic velocity to

$$\dot{x} = \frac{F}{\mathfrak{z}'} = \frac{F}{\mathfrak{z}_d + \mathfrak{z}} \qquad \text{rms. kines} \angle \qquad (11)$$

or

$$\mathfrak{z}' = \frac{F}{\dot{x}} \qquad \text{total mechanic absohms} \angle \qquad (12)$$

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2 Bibliography 9, page 477.
3 Bibliography 7, 8 and 9.

$\mathbf{z}'$ contains an internal mechanic impedance $\mathbf{z}_d$, and an external acoustic impedance $\mathbf{z}$, including both surfaces of the diaphragm. If the receiver is operated *in vacuo*, the external acoustic impedance $\mathbf{z}$ disappears, and the total mechanic impedance on the diaphragm is reduced to $\mathbf{z}_d$.

The rms. velocity $\dot{x}$ of a telephone receiver diaphragm, and also the impressed vmf. F, can be measured electrically in the laboratory by mapping out the motional impedance diagram, and measuring the amplitude of vibration at resonance. Consequently, by means of (12) the total mechanic impedance on the disk can be evaluated. If we measure the internal mechanic impedance, we can then find $\mathbf{z}$ the acoustic impedance, and ascertain how it varies under different conditions. The measurements described in this paper offer a starting point in this direction.

Brief History of the Method: In preceding researches on the motional impedance of telephone receivers, the receiver has been usually regarded as a motor, and the mechanic resistance of the receiver diaphragm as a motor load, partly due to internal frictions, and partly to external acoustic wave emission. In a few cases, the mechanic resistance was varied, either by operating the receiver in a partial vacuum, or by attaching to the diaphragm a disk vane immersed in a damping fluid.[4] In the research here described, however, the results obtained have led to changing the function of the tested receiver from a reciprocating motor to a reciprocating generator of sound, with the object of studying the reaction, or change of acoustic load, on this generator, produced by sound reflection and absorption. It is believed that the method here described is susceptible of numerous applications in the science of acoustics. Although the definition of acoustic impedance offered in this paper is believed to be new; yet the concept can be discerned in the writings of various acousticians.[5]

Outline Theory of the Method: It has been shown in preceding publications, that if A is the force factor of a telephone receiver in dynes per absampere $\angle$, a complex or planevector quantity having the lagging slope $\beta°$, and representing the electro-magnetic pull upon the diaphragm per rms. absampere of exciting current I, taken at standard phase, Z' is the motional impedance of the receiver as measured at the impressed frequency, and with a given acoustic load (absohms

4 Bibliography 9, pages 460–462.

5 Bibliography 15. Since delivering this paper the authors' attention has been directed to Dr. Drysdale's recent Kelvin lecture (Bibliography 20) where this theory is developed.

$\angle$), $\mathbf{z}'$ is the mechanic impedance of the diaphragm at the same frequency and load (dynes per kine $\angle$ or mechanic absohms $\angle$). Then $\dot{x}$ the rms. vibrational velocity of the diaphragm over either pole of the receiver is

$$\dot{x} = \frac{\mathrm{A\,I}}{\mathbf{z}'} \qquad \text{rms. kines } \angle \qquad (13)$$

If the mechanical impedance $\mathbf{z}'$ is reactanceless, as happens at the impressed frequency of apparent resonance, it degrades from a complex quantity to the real quantity r'', or mechanic resistance. In this case, the phase of the velocity $\dot{x}$ will be the same as that of the product (AI). Taking the phase of I as standard, or its slope as zero, that of A is $-\beta°$; so that $\dot{x}$ will then lag $\beta°$ behind I in phase. In general, however, with mechanic reactance present, the phase of $\dot{x}$ will be displaced from that of (AI) by the slope of $\mathbf{z}'$.

Again,

$$\mathrm{Z}' = \frac{\mathrm{A}\dot{x}}{\mathrm{I}} = \frac{\mathrm{A}^2}{\mathbf{z}'} \qquad \begin{matrix}\text{electric absohms } \angle \\ \text{or C.G.C. magnetic} \\ \text{units of resistance } \angle\end{matrix} \qquad (14)$$

This means that in the motional impedance circle of the receiver, at any impressed frequency, the vector motional impedance Z′, as obtained from electrical measurements with a Rayleigh bridge, is equal to the square of the complex force factor A, divided by the total mechanic impedance $\mathbf{z}'$ at that frequency. At the frequency of apparent resonance, this becomes

$$\mathrm{Z}'_\circ = \frac{\mathrm{A}^2}{r''} \qquad \text{electric absohms } \angle \qquad (15)$$

since $\mathbf{z}'$ degrades at resonance into the total mechanical resistance r''. The slope of the motional impedance Z′ will then be the same as the slope of A^2, or $-2\beta°$. $\mathrm{Z}'_\circ$ is thus the diameter of the motional impedance circle, in the ordinary simple case, where the telephone is tested for its motional impedance.

From (14) it follows that, at any impressed frequency, the total mechanic impedance $\mathbf{z}'$ is

$$\mathbf{z}' = \frac{\mathrm{A}^2}{\mathrm{Z}'} = \mathrm{A}^2\,\mathrm{Y}' \qquad \begin{matrix}\text{dynes per kine } \angle \\ \text{or mechanic absohms} \angle\end{matrix} \qquad (16)$$

Consequently, if we divide the complex constant A^2 by the measured motional impedance Z′, we obtain the size and slope of the total mechanic impedance of the diaphragm at this frequency.

The total mechanic impedance $\mathbf{z}'$ is made up of several mechanic impedances; namely,

(1) The mechanic impedance of the diaphragm

$$\mathbf{z}_d = r_d + j\left(m_d\omega - \frac{s_d}{\omega}\right) \qquad \text{mechanic absohms } \angle \quad (17)$$

(2) The virtual mechanic impedance of the diaphragm [6] due to its motion in the permanent magnetic field

$$\mathbf{z}_v = j\,\frac{p\mathrm{A}}{\omega} = r' + \frac{js'}{\omega} = \frac{p\,|\,\mathrm{A}\,|}{\omega}\sin\beta + j\,\frac{p\,|\,\mathrm{A}\,|}{\omega}\cos\beta$$

mechanic absohms $\angle$ (18)

The real part of $\mathbf{z}_v$, although varying inversely with ω, is ordinarily taken as constant to a first approximation, the working range in ω being usually small.

(3) The acoustic impedance $\mathbf{z}_1$ of the air in the chamber behind the diaphragm, and which varies, in some manner similar to (17), as the impressed frequency is varied.

(4) The acoustic impedance $\mathbf{z}_2$ of the air in front of the diaphragm, and which constitutes the load to be varied.

Consequently,

$$\mathbf{z}' = \mathbf{z}_d + \mathbf{z}_v + \mathbf{z}_1 + \mathbf{z}_2 \qquad \text{mechanic absohms } \angle \quad (19)$$

Keeping the impressed frequency constant, $\mathbf{z}_d$, $\mathbf{z}_v$, and $\mathbf{z}_1$ will remain constant; but $\mathbf{z}_2$ can be made to vary by changing the acoustic load. The changes in the electric impedance, measured on changing the acoustic load, enable the latter to be computed. Moreover, by observing the changes in Z′ when the frequency is varied, if allowance can be made for the changes thereby produced in $\mathbf{z}_d$, $\mathbf{z}_v$ and $\mathbf{z}_1$, the variations in the acoustic impedance $\mathbf{z}_2$ may also be computed.

Measurement of Acoustic Impedance of the Air Column in a Tube of Varied Length: A straight smooth fibre tube 87.8 cm. long, 3.1 cm. internal diameter, and 3.9 cm. external diameter, was supported horizontally, by metallic strips packed with felt, over a wooden base BB, Figure 2. The tube TT was brought into contact with the cover of a regular form of bipolar telephone receiver R, as is shown in detail by Figure 3. The axis of the tube was in line with the axis of the receiver. A hard rubber piston PP slides freely in the tube, and can

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[6] Bibliography 16.

be set at any desired position within it, by means of the projecting brass rod *r*. The telephone receiver R is clamped, by means of a metallic strip, on to a wooden pedestal rising from the base BB.

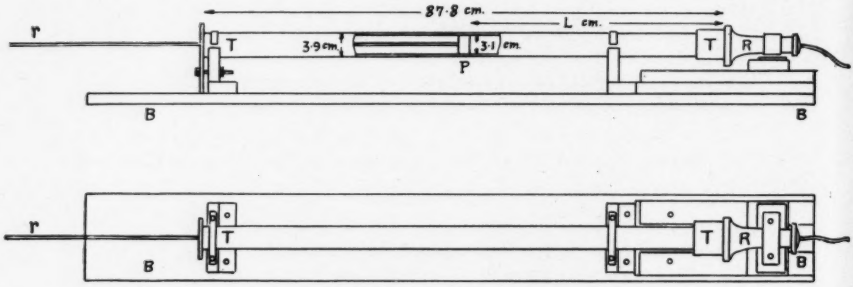

FIGURE 2. Side Elevation and Plan of Experimental Tube.

The telephone receiver used in the tests here described was a bipolar Western Electric Company's instrument of a well known pattern, provided with a composition cap or cover. This cover had its opening enlarged, as is shown in Figures 2 and 3, so as to connect more easily with the acoustic tube TT.

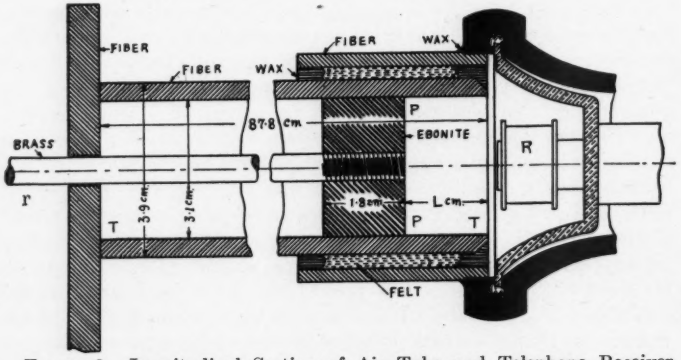

FIGURE 3. Longitudinal Section of Air Tube and Telephone Receiver fastened to the Same.

The electric impedance of the receiver was measured in the Rayleigh bridge shown in Figure 4. The adjustable-frequency source S, was a

Vreeland oscillator in some of the tests, and a triode vacuum-tube oscillator in others. The resistance R was adjusted to give an alternating current of 3 rms. milliamperes to the bridge. In some tests, this current was raised to 4 milliamperes. Half this current passed through the receiver Z, and the other half through the adjustable anti-inductive resistance *r*, and variometer *l*. The two arms *a* and *b* of the bridge are of equal anti-inductive resistances. The current-detector, or galvanometer, was a pair of medium-resistance head telephones.

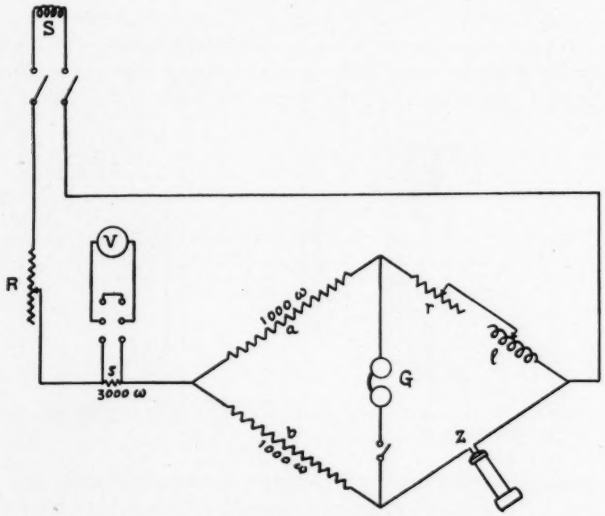

FIGURE 4. Electric Connections of Rayleigh Bridge for measuring the Motional Impedance of the Telephone Receiver Z.

The measurements were all made with approximately constant *a–c.* strength through the measured receiver Z. The measured impedance of the receiver did not, however, vary materially over a small range in current strength; so that it was not important to keep the current strength very nearly constant. The current was ascertained and adjusted by means of a static voltmeter V, bridged across an anti-inductive resistance *s* of 3000 ohms. In some of the tests, a thermojunction and *d-c.* galvanometer were substituted for *s* and V.

Technique of Measurements: A series of measurements is first made of the apparent resistance R and inductance L of the receiver **z**, Figure 4, over a fairly considerable range of frequency; so as to include the resonant frequency f_o of the receiver, which is allowed to vibrate in the free air of the testing room. This series of measurements of R and L at varied frequency, but constant testing-current strength, is then repeated without delay, with the diaphragm damped, or prevented from vibrating. This damping may be effected in several ways, that have been described in preceding publications.[7] A convenient procedure, however, has been found to consist in forming a T of paraffin wax, and melting the extremities of this with a hot wire, so as to press gently, but adherently, on the center of the diaphragm, and also across a diameter of the receiver cap, as is indicated in Figure 5. By this means, the diaphragm is prevented from vibrating without altering the normal airgap. When great precision is not desired, a still swifter and simpler method of damping the receiver, is to insert a round plug of wood, wax, or other solid material, into the hole in the receiver cap, so as to plug this hole without touching the diaphragm. This method of damping the receiver has been called "acoustic damping." It permits of some vibration of the diaphragm, especially towards higher impressed frequencies; but the cushioning of the air, by the inserted plug, prevents the vibration from exceeding a small fraction of that which occurs with the plug withdrawn. In other words, the acoustic impedance $\mathbf{z}_2$ of such a plug and air cushion is very large.

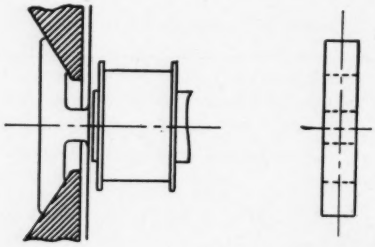

FIGURE 5. Diagram indicating the mode of clamping a Receiver Diaphragm by the application of a Wax Bridge or T.

7 Bibliography 7, 9.

The "damped impedance" of the receiver is subtracted vectorially from the "free impedance," for a number of successive frequencies. The differences, or "motional impedances" of the receiver are plotted on a separate diagram. The ordinary motional-impedance diagram of a receiver, thus tested in the open, is very nearly a circle passing through the origin; i. e., the well known motional-impedance circle. The motional-impedance circle of the tested receiver is given in Figure 6.

Its diameter is $158 \angle 55.^{\circ}6$ ohms; so that the angle $\beta^{\circ} = -\frac{55.^{\circ}6}{2} = -27.^{\circ}8$.

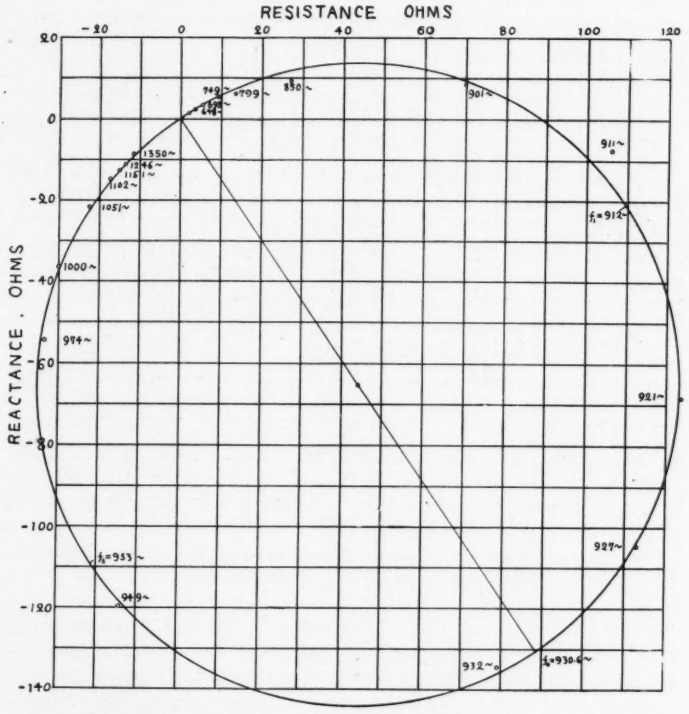

FIGURE 6. Motional Impedance Circle of Telephone Receiver Used in the Tests.

The *d-c.* resistance of this receiver at 20°C is 86.4 ohms; so that the motional impedance per unit of *d-c.* resistance, Z'_o/R_1 for this instru-

ment is [8] $1.83\angle 55.6°$. The angle $\beta°$, according to theory, is the angle which the locus of damped impedance makes with the vertical or X axis, in the neighborhood of the resonant frequency. The frequency of apparent resonance is $f_o = 930.6\,\omega$ at 20°C. The quadrantal frequencies are $f_1 = 912\,\omega$ and $f_2 = 953\,\omega$; so that $\Delta = 41\pi = 128.8$ and the sharpness of resonance $\Lambda_o = 2\pi f_o/\Delta = 45.4$.

The next step is to connect the receiver to its acoustic load. In the case of Figure 2, the load is the cylindrical column of air in the tube TT. The resistance and inductance of the telephone are now meas-

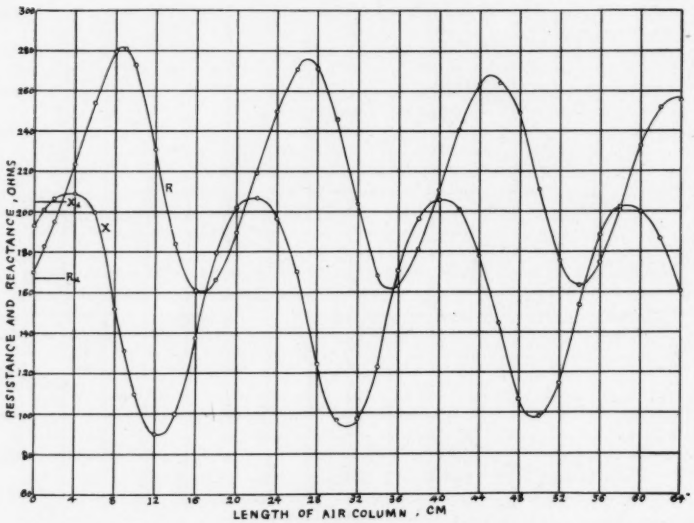

FIGURE 7. Curves of Apparent Resistance and Reactance of Receiver when attached to Air Column of successively varied Length.

ured at successive positions of the piston P, holding the frequency f, and alternating-current strength, constant. The values of receiver resistance and inductance so measured, are found to fluctuate periodically as the length L of the air column in the acoustic load changes. Figure 7 gives a pair of wavy curves in one test, at $f = 921\,\omega$ and I = 1.575 milliamperes rms. in the receiver.

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8 Bibliography 16, page 527.

It is important that the frequency and the temperature in such a test should be held as nearly constant as possible. The frequency should be near to the resonant frequency f_o. If the frequency is remote from the resonant frequency, the mechanic reactance of the diaphragm is likely to be so large that the addition of the acoustic load to be measured will not affect the vector motional impedance materially; whereas at resonance, a small change in added acoustic load will be likely to affect the motional impedance considerably.

The variations in R and X indicated in Figure 7, represent corresponding variations in the electric impedance $Z = R + jX$. These are due to variations in the acoustic impedance $\mathfrak{z}_2$, which is the subject of investigation. The curves of R and X in Figure 7, represent total apparent resistance and reactance, in the receiver, at varying air-column lengths. The values R_d and X_d indicated on the left-hand side of the figure, are respectively the damped resistance and reactance of the receiver at this frequency. With reference to the impedance $Z_d = R_d + jX_d$, as origin, the values of motional impedance Z', corresponding to the successive air-columns, are plotted vectorially in Figure 8. The numbers marked on this spiral are air-column lengths in cm. It will be seen that at L = 0 cm., or with the piston within 1.5 mm. of the diaphragm, the motional impedance was $12.5 \angle 75°.4$ ohms. This value was nearly repeated with L = 19 cm. At L = 10 cm., however, the motional impedance had increased to $142.8 \angle 42°.1$ ohms. The curve is a slowly contracting spiral, with a pitch of 18.7 cm., which is half a wave length at 921 ∞ ; since the velocity of sound at 20°C being 34,430 cm. per sec., the wave length $\lambda = 34{,}430/921 = 37.38$ cm.

In order to utilize the motional-impedance diagram of Figure 8 for determining the acoustic impedance of the load in front of the receiver diaphragm, we may refer to equations (16) and (19). Equation (16) shows that we must find Y', the vector reciprocal of the motional impedance, and multiply by the square of the vector constant A of the receiver, in order to derive the total mechanic impedance $\mathfrak{z}'$ on the diaphragm. This means that we must invert the diagram of Figure 8, or find the locus of the reciprocal spiral.

If we operate upon the graph of motional admittance by the vector factor A^2, where $A = 5.138 \times 10^6 \angle 27°.8$ (the value measured for this particular receiver)[9]; i.e., multiplying each vector admittance

[9] In order to determine the value of the vector force factor A, it is necessary to measure the amplitude of receiver diaphragm vibration at resonance, as has been described in preceding papers, Bibliography 9.

by $26.4 \times 10^{12} \angle 55°.6$, we obtain the total mechanic impedance graph of Figure 9. Here the curve is carried from L = 0 cm. to L = 62 cm. Thus in Figure 8, the vector 0–18 measures $26 \times 10^{9} \angle 92°.6$ absohms. The reciprocal of this is $Y' = 0.03846 \times 10^{-9} \angle 92°.6$ abmhos. Multiplying this reciprocal by $A^2 = 26.4 \times 10^{12} \angle 55°.6$,

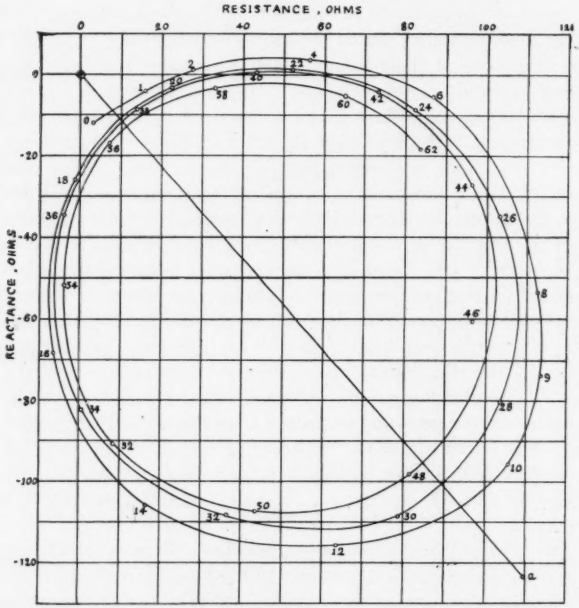

FIGURE 8. Motional Impedance Diagram of Receiver at Constant Impressed Frequency (921 ∞) with varying length of Attached Air Column.

we obtain $1.015 \times 10^{3} \angle 37°$ dynes per kine. This is the vector total mechanic impedance, at 18 cm. of air-tube length. It will be observed that the curve is a spiral, with the same direction of rotation as in Figure 8, but the relative positions of observation points about the axis have been changed. In Figure 9, the straight line A B C D E is the axis of the acoustic impedance spiral.

Analysis of Figure 9 indicates that the acoustic impedance OA = $160 \angle 7°.5$ mechanic absohms is the total internal mechanic imped-

ance ($\mathbf{z}_d + \mathbf{z}_v + \mathbf{z}_1$) at the impressed frequency of the measurement (921 $\sim$). This leads to the inference that if the acoustic impedance in the tube and in front of the receiver diaphragm could be completely removed, the mechanic impedance left in and behind the diaphragm would be the vector impedance OA. This would correspond to a vector motional impedance Oa = 165 $\angle$ 48°.1 on Figure 8. Oa is therefore the inferred motional impedance for the case of removal of all acoustic load from the front of the diaphragm. The addition of the acoustic load of the air column in the tube is a vector

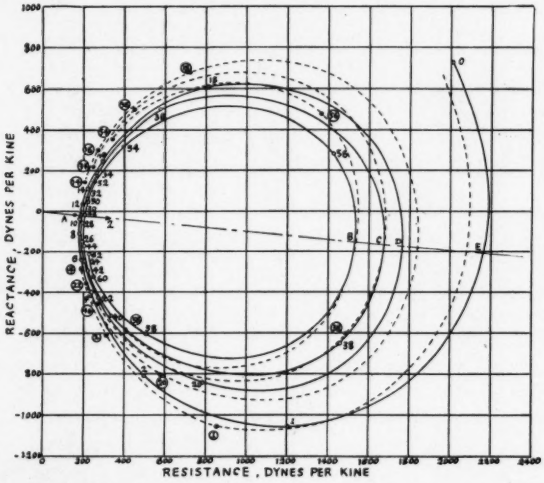

FIGURE 9. Graph of Total Mechanic Impedance on Receiver Diaphragm obtained by inverting the graph of Figure 8 and multiplying by A^2. Heavy curve connects observation values. The dotted curves represent the computed values.

with its origin at A, Figure 9. The spiral locus 0 1 2 4. . . . 62 is the spiral of vector impedance of the tube acoustic conductor, with its distant end sealed at distances of 0 1 2 4. . . . 62 cm. from the diaphragm, or more strictly from a plane 1.5 mm. in front of the diaphragm, where the tube may be considered as beginning. This spiral impedance conforms fairly well with the familiar expression

$$\mathbf{z}_o \tanh \delta_A \qquad \text{dynes per kine} \angle \qquad (20)$$

where z_o is the "surge impedance" of the tube, or the acoustic imped-

ance offered to this frequency by an indefinitely great length of the tube, and δ_A is the "position angle" at the sending end A of the tube as the length of the same is varied. The value deduced for $\mathbf{z}_0$ is $187 \angle 5°.2$, or A Z on Figure 9. This point Z is thus the inferred inner terminus of the spiral E D C B if the tube were prolonged indefinitely. The hyperbolic angle δ_A appears to be

$$\delta_A = \{L\,(0.000706 + j\,\underline{0.107}) + (0.0974 + j\,\underline{0.97})\} \text{ hyps } \angle \quad (21)$$

In the electric-conductor analogy, if the tube were stopped with a perfectly reflecting air-tight plug, the position angle at the point of stoppage would be $j\frac{\pi}{2}$ hyperbolic radians or $j\,\underline{1}$; i.e., one imaginary quadrant,[10] or 90°. This imperfection of the plug apparently alters this to $0.0974 + j\,\underline{0.97}$.

The dotted spiral in Figure 9 indicates the computed locus according to (21). The agreement between the heavy spiral of observations and the dotted spiral of formula (21) is seen to be fairly satisfactory. At the time when the observations were made, the theory of the analogy between the impedance of an acoustic-tube conductor and that of an electric-line conductor had not been reached. It has been pointed out, however, by Fleming [11] and Fitzgerald [12] that an analogy exists between the waves along air tubes and electric waves.

According to this analogy, therefore, the plugged tube of varied length offered an acoustic impedance similar to the electric impedance of a uniform alternating-current line, of varied length, put to ground in each case at the distant end through a certain leak, or high resistance. At each quarter wave of tube lengthening, the impedance of the tube crosses the axis of $\mathbf{z}_0$ and reaches alternately a high and a low value. Thus at C = 37.7 cm., the impedance is a maximum:

$$\begin{aligned}\mathbf{z}_{37.7} &= 187 \angle 5°.2 \tanh \{(0.0266 + j\,\underline{4.03}) + (0.0974 + j\,\underline{0.97})\} \\ &= 187 \angle 5°.2 \tanh (0.124 + j\,\underline{5}) \\ &= 187 \angle 5°.2 \coth 0.124 = 187 \angle 5°.2 \times 8.106 = 1517 \angle 5°.2,\end{aligned}$$

acoustic absohms $\angle$

The quarter wave length is 9.35 cm.; so that at L = 0.30, 19.0, 37.7, 56.4 cm., the acoustic impedances should be maxima, and should fall on the axis of $\mathbf{z}_0$ at E, D, C and B respectively; while at L = 9.65,

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10 Bibliography 11 and 13.
11 Bibliography 12.
12 Bibliography 2.

28.35, 47.05 cm., the impedance should be a minimum, and should fall on the same axis, nearer to A. All these requirements are fairly well met in the diagram of Figure 9.

It is thus possible, by making purely electrical measurements of the motional impedance of a telephone receiver, to arrive at the corresponding variations in acoustic impedance, when the acoustic load is varied, at constant impressed frequency. In the case of a simple air tube of varied length, the acoustic impedance appears to follow substantially the same quantitative behavior as the electric impedance of an alternating-current line of correspondingly varied length.

In the case of an electric line, we can always determine the values of the line angle θ and the surge impedance z_0, at a point, by successively grounding and freeing the line at the distant end, and measuring the corresponding impedance $Z_{\theta A}$ and Z_{fA} at the home end A. The ratio of these two impedances is $\tanh^2\delta_A$ and their product is z_0. In the case of a tube or acoustic line, we can approximately free the line, by plugging the tube at the far end with a smooth hard plug, which is designed to serve as a perfect sound reflector; but no means are at present available for "grounding" the acoustic line. With the tube wide open at the distant end, there is still an appreciable acoustic impedance in and beyond the open end. If the tube opens into a room or walled space, this terminal acoustic impedance will, in general, possess some reactance due to reflections from the walls; but if the tube terminates in the open air, the terminal acoustic impedance is likely to have only a small reactive component or slope. If the amount of the terminal acoustic impedance σ of a tube opening out of doors, could be ascertained from the geometry of the tube, it would be readily possible to compute θ and $\mathbf{z}_0$ from the acoustic impedance Z_{fA} with the distant end plugged, and $Z_{\sigma A}$ with the distant end open to the free air. Until σ can be satisfactorily predetermined, it will be necessary to measure θ and $\mathbf{z}_0$ by indirect electrical methods, and to determine σ from the differences between Z_{fA} and $Z_{\sigma A}$. Such investigations might lead to the experimental determination of σ, the open-end terminal acoustic impedance of a tube of given dimensions.

Effects of different Apertures at the Far End of the Tube on the Acoustic Impedance at the Sending End: Figure 10 shows several motional impedance diagrams for the case of a fixed length of the same air-tube (87.8 cm.), each diagram being taken at constant impressed frequency, and with different sizes of circular aperture in the plug at the distant end of the tube. The tube was closed with a flat slab of fiber 0.62 cm. thick. Circular holes marked Nos. 1, 2, 3, and 4 were

cut in different parts of this slab, with the following diameters; No. 1, 0.6 cm.; No. 2, 0.95 cm.; No. 3, 1.3 cm.; No. 4, 1.6 cm., and No. 5, full tube aperture (3.1 cm.); i.e., with slab removed. No. 0 represents the case of a complete closure of the tube by the unpierced slab.

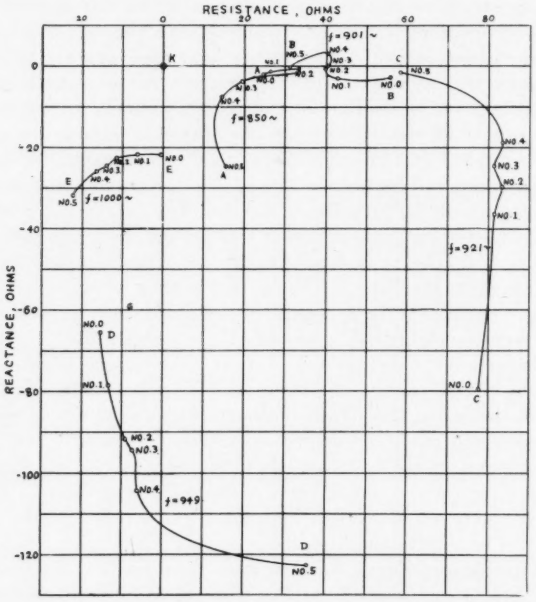

FIGURE 10. Motional Impedance of Receiver at Constant Impressed Frequencies, attached to Air-Tube of Constant Length (87.8 cm.) with apertures of different diameters at far end.

The origin of the diagram is at K, Figure 10. At the constant frequency of 850∞, the successive motional impedances of the receiver attached to the tube, with successive terminal orifices 0–5 were K–O, K–1. . . . K–5. The ends of these vectors are connected by the curve AA. Repeating the motional-impedance measurements at the successive steady frequencies of 901, 921, 949 and 1000∞, the corresponding vector loci with different terminal apertures are marked on the curves BB, CC, DD, and EE.

It is evident that the motional-impedance variations, due to varying the distant-end orifice of the tube, are greatest on the long curves CC and DD, apparently because the frequencies pertaining to them (921∞ and 949∞ respectively) lie nearest to the resonant frequency of the receiver, when loaded with the tube under these conditions. For sensitiveness in the measurement of tube terminal effects in the receiver, the frequency of 850∞ was too low, and that of 1000∞ too high.

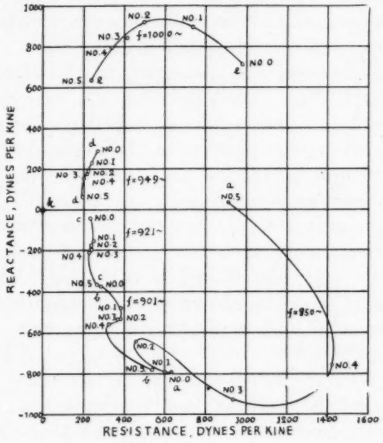

FIGURE 11. Total Mechanic Impedance at Receiver Diaphragm by Inversion from Figure 10 and Multiplying by A^2.

The mechanic impedance at the receiver diaphragm, in the various measurements recorded in Figure 10, are presented in Figure 11, after inversion and multiplication by A^2. These mechanic impedances are vectors, expressible in mechanic resistance and reactance, as dynes per kine, or mechanic absohms, as may be preferred. The origin in Figure 11 is at *k*. The vector graph A A of Figure 10, becomes converted into the vector graph *a a* of Figure 11, B B into *b b*, and so on.

It will be observed that the most sensitive graphs C C and D D of Figure 10 give rise to graphs *c c* and *d d* in Figure 11, revealing only small changes in mechanic resistance, but relatively large changes of mechanic reactance, with change of tube aperture. On the other hand, *e e* at 1000∞ includes more change in mechanic resistance

(230 to 980 dynes per kine) than in mechanic reactance (j 630 to j 920 dynes per kine). This indicates that the changes in acoustic load, effected by altering the tube aperture at the distant end, may preponderate in mechanic resistance, or in mechanic reactance, depending upon the impressed frequency, and the system of stationary sound waves set up in the tube.

Another set of measurements was made with the same telephone receiver and fiber tube plugged at various tube lengths, but with a row of holes, like those of a flute, bored in the tube wall. The diameter of each hole was 6.5 mm., and their distance apart, between centers, 4 cm. Removable plugs were inserted in these holes. The motional impedance of the receiver was measured, at four steady frequencies between 900∞ and 1000∞, when one plug was removed at successive holes along the line. The motional impedance was found to be sensitively affected by the position of the particular plug removed. The results, although interesting, have not been analyzed, and are therefore not presented here in quantitative detail. They indicate, however, that the acoustic impedance of the air column in a tubular musical instrument, such as a flute, undergoes marked variations when the instrument is manipulated, as in playing. These changes of acoustic impedance might be measured, in the manner described, with respect to a telephone-receiver diaphragm as the source of sound.

Motional Impedance and Acoustic Impedance of a Telephone Receiver pressed against the Ear: When a telephone receiver is exposed to the air in a large room, the total mechanic impedance on its diaphragm is the total internal impedance ($\mathbf{z}_d + \mathbf{z}_v + \mathbf{z}_1$) plus the external impedance $\mathbf{z}_2$ of the air in front of the diaphragm, including the cushion of air under the cap. The ordinary motional impedance circle of the instrument is measured under these conditions. When, however, the telephone is pressed against the ear, the external acoustic impedance $\mathbf{z}_2$ is that of the air in the ear cavity, as well as in the cap of the instrument. The motional impedance of the instrument under these conditions, approximates to that actually presented in the average telephonic use of the device as a receiver of speech.

Figure 12 gives the motional impedance circle O A B C of a bipolar receiver, No. 143–2, similar to that used in the tests recorded in preceding figures. The frequency of apparent resonance was $f_\circ = 969\infty$ when the motional impedance was $177\angle 44^\circ.2$ ohms. This receiver, when held against the ear of a listener, as in the ordinary conversational use of the instrument, gave rise to the smaller graph O D E F, which departs very clearly from the strictly circular form. The exact

shape of this graph O D E F is not regarded as of great importance; because, if the instrument was placed against the ear with less or more pressure, or if it was centered differently on the ear; or if some other ear was selected, the shape of the graph would be likely to be altered appreciably. To a first approximation, however, the graph O D E F may be regarded as an approach to the dotted circular motional impedance O G E H, which may be described as the motional circle of

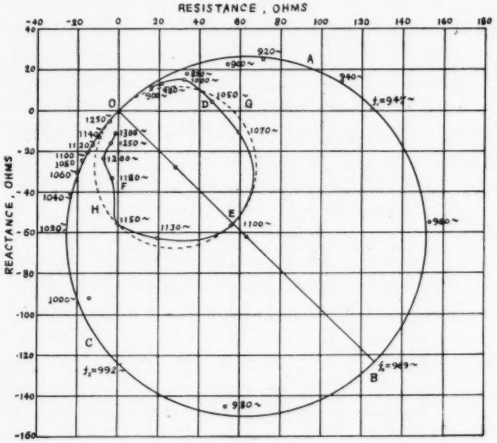

FIGURE 12. Motional Impedance Graphs of Receiver in Room and pressed against Listener's Ear.

reference, with the instrument applied to this ear. It will be observed that the diameter of this circle of approximation is $78.5 \angle 44°$ ohms. The diameter O E has, therefore, substantially the same slope as the free-air motional impedance diameter O B; but has been reduced in length somewhat more than 50 per cent. The frequency of apparent resonance has also been changed from 969 ∞ to 1100 ∞, by applying the receiver cap to the ear.

Figure 13 gives the corresponding mechanic impedance graphs for this receiver and ear. The straight line *a b c*, parallel to the reactance axis, corresponds to the motional impedance circle O A B C of Figure 12, taken in the free air of a room. This means that the total mechanic impedance of the receiver free to the room, was a constant mechanic

resistance of 149 dynes per kine, plus or minus a mechanic reactance, depending on the impressed frequency. At 969∞, the frequency of apparent resonance, this reactance disappears, leaving the resistance of 149 dynes per kine as the residual mechanic impedance.

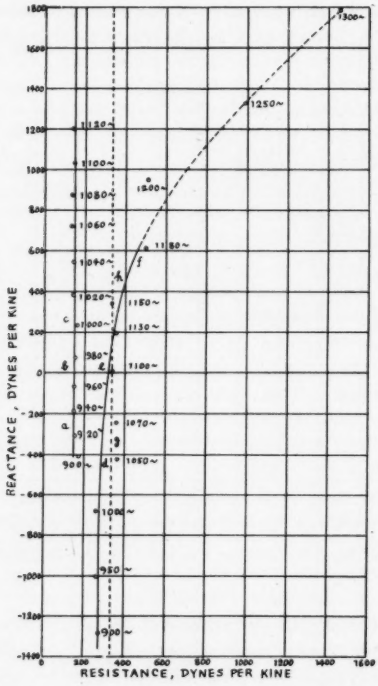

FIGURE 13. Mechanic Impedance of Receiver in Room and when pressed against Listener's Ear, by inversion from Fig. 12, and multiplying by A^2.

It should be observed that any motional-impedance circle of a telephone receiver, passing through the origin of coördinates, corresponds to such a straight-line graph of total mechanic impedance parallel to the reactance axis, no matter what the depression angle of the motional-impedance diameter may be. The application of the factor A^2, in formula (16), cancels out the diametral depression angle.

In other words, all motional-impedance circles of telephone receivers are convertible into rectilinear mechanic impedance graphs of constant mechanic resistance, neglecting the influences which are known to distort the motional impedance graph from the purely circular form.[13]

If the motional-impedance circle of approximation O G E H of Figure 12 is accepted for the receiver applied to the listener's ear; then the dotted straight line *g e h*, Figure 13, would represent the corresponding mechanic impedance graph, where the mechanic resistance is 335 dynes per kine, and zero reactance is found near 1100∞. The curved line *d e f* corresponds more nearly to the actually recorded motional-impedance graph O D E F, Figure 12. The particular shape of this curved graph *d e f* may not, however, be relied upon.

Figure 13 indicates therefore that, at least to a first approximation, the acoustic resistance of the listener's ear in this measurement was 86 dynes per kine. At higher frequencies, this resistance appears to increase.

Experiments were also made with an artificial ear,[14] composed of paraffin wax, and in form modelled closely after an anatomical model human ear. A motional-impedance graph made for the same telephone receiver, applied to this artificial ear, agreed fairly well in form with the graph for the actual ear O D E F, Figure 12. The corresponding approximate circle had a diameter of $98 \sphericalangle 44°$ ohms; or was somewhat larger than the circle O G E H of Figure 12, corresponding to a somewhat lower mechanic resistance. It may be noted that a receiver, when applied to a model ear in a solid substance like paraffin wax, does not fit to it as closely as when applied to the human ear. The imperfect fit would tend to lower the acoustic resistance.

With the receiver applied face downwards upon a flat slab of glass, a motional impedance graph was a closed curve through the origin roughly circular in form. The approximating equivalent circle had a diameter and depression angle about the same as when the measure-

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13 These deviating influences are the variation in the force factor A, with impressed frequency, and the variation in r' the component of hysteretic frictional resistance of the diaphragm, with change of impressed frequency; see Bibliography 16.

14 It is believed that an "artificial ear," consisting of a flat slab of some suitable substance selected experimentally, on which a receiver is laid face down, has been used for some years in the telephone laboratories of the Western Electric Co. in New York.

ment was made with the receiver open to the air of the room. The frequency of apparent resonance was, however, raised to 1090 ∞.

Application of the Method to the Tuning of Telephone Receivers for increasing their Sensibility: Referring to Figures 7 and 8, it will be seen that for the given impressed frequency of 921 ∞ on the receiver connected to the fiber tube, varying the length of the tube through a range of 9.4 cm., or one quarter of a wavelength, altered the motional impedance from a maximum to a minimum, or *vice versa,* and also altered the maximum cyclic velocity of the diaphragm in the ratio of more than 8:1. From this it will be evident that when such a tube is inserted between the telephone cover and the listener's ear, by the insertion of an ear tube into the plug P, Figure 2, the loudness of the sound emitted by the receiver, under steady excitation, can be adjusted by altering the tube length. In other words, the tube length can be tuned to maximum sensibility of the receiver. This principle may have useful applications in measurements with the telephone receiver as a detector. Observations made in this way have shown that this method of tuning is applicable over a certain range of impressed frequency. Thus, if the impressed frequency is 900 ∞, and the receiver has an apparent resonant frequency, under its actual acoustic load, of 1000 ∞, an inserted adjustable tube may enable the resonant frequency to be reached at 900 ∞.

Possible Applications of the Method to Architectural Acoustics: When the free impedance of a telephone receiver is measured with the axis of the instrument horizontal, and with its cover facing a wall of the room at not too great a distance, it is found that, near the frequency of apparent resonance, the electric impedance of the instrument is affected by its distance from the wall. Holding the frequency steady, the electric impedance of the receiver is found to vary if the instrument is moved nearer to, or farther from, the wall which it faces. This means that the acoustic impedance, at the diaphragm, of the air in front of the instrument, is affected by the distance from the wall in relation to the wave length of the emitted sound. The amount of variation of electric impedance, or the sensibility of the measurement to small mechanical displacements, depends upon the sharpness of resonance of the diaphragm. It seems likely that this method of measurement is capable of being applied to architectural acoustics; as, for example, in the measurement of the sound-reflection coefficient of draperies. For such purposes, a telephone receiver should be selected with a small total mechanic resistance in its diaphragm, and with a small damping constant Δ; i.e., with a large sharpness

of resonance.[15] Such an instrument would have a relatively large motional-impedance circle, and a small change of impressed frequency would carry the vector from one quadrantal point, through resonance, over to the other. Relatively feeble reflected sound waves, falling on the diaphragm, would thus appreciably affect the motional impedance, and enable the corresponding acoustic impedance variation to be measured.

Summary of Results.

(1) Acoustic resistance, reactance and impedance are defined. A technique for their measurement is described.

(2) Acoustic impedance is offered to any disk or diaphragm steadily vibrating in a fluid.

(3) Acoustic impedance can be measured at a telephone-receiver diaphragm, but can be considered as existing in a fluid at any surface that can be drawn, and across which sound is steadily transmitted.

(4) When the amplitude of vibration of a diaphragm is not uniform over its surface, the acoustic activity over its surface can be evaluated in terms of the mean square amplitude over the surface.

(5) Acoustic impedance at the diaphragm of a telephone receiver, carrying an alternating current, can be determined by measuring changes in the motional impedance of the receiver under changes of acoustic load.

(6) The total mechanic impedance at a receiver diaphragm, under any steadily impressed a–c. frequency and acoustic load, is the reciprocal of its motional impedance, multiplied by A^2, the square of the vector forcefactor of the instrument.

(7) The mechanic impedance graph, under varied frequency, of any receiver whose motional impedance diagram is a circle through the origin, consists of an infinite straight line parallel to the reactance axis.

(8) By applying a telephone receiver to one end of an air tube, or to the mouth of any air chamber, the acoustic impedance of the tube or chamber can be determined in this condition, by measuring its effect upon the motional impedance in the neighborhood of the resonant frequency.

(9) In the case of a long straight cylindrical air-tube with smooth walls, attached to the cap or cover of a receiver, the stationary sound-

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15 Bibliography 7, page 133 and Fig. 11, where the receiver had a resonant sharpness of about 320.

wave system set up in the tube, at a given frequency, appears to correspond to the stationary electromagnetic wave system set up in a certain equivalent alternating-current line conductor. The quantitative relations in the two cases present remarkable analogies.

(10) The plugging of the air tube at its distant end in the last mentioned case, corresponds to freeing the a–c. electric conductor at its distant end. On the other hand, although the electric conductor can be grounded at its distant end, no corresponding means appear to be available for reducing to zero the acoustic impedance of the open end of the tube.

(11) Applying a telephone receiver to the ear of a listener roughly doubled the total mechanic resistance at the receiver diaphragm, and also increased its apparent elastic constant.

(12) A telephone receiver may be tuned to maximum response or vibratory displacement, over a certain range of impressed frequency, by altering the length of an air column between the receiver and the listener's ear.

(13) The method of measuring acoustic impedance electrically appears to be applicable to architectural acoustics.

(14) Tubular musical instruments, such as flutes, evidently possess acoustic impedances that are variable under manipulation, and that are susceptible of measurement under the particular conditions described.

Appendix.

On the Analogies between Sound-Wave Transmission along a Uniform Acoustic Tube Conductor, and Electromagnetic-Wave Transmission along a Uniform Electric Line Conductor.

We may assume that when a telephone receiver, actuated as an acoustic generator, sings a pure tone steadily into a long smooth straight tube of S sq. cm. internal cross section, arranged as in Figure 2, the sound waves travelling along the tube are plane waves, and that the effects on them of friction at the walls of the tube may be neglected. Even if the actual conditions do not conform closely with these postulates, the results thus arrived at may safely be regarded as first approximations.

The distance of any cross-section of the tube, measured along the axis, from the generator diaphragm may be expressed as **l** cm. The clamping-circle diameter of the receiver diaphragm may first be supposed to be equal to the internal diameter of the tube. The very small vibratory displacement of the air, in a sound wave, from its normal quiescent position, measured parallel to the axis, in the direction of increasing **l**, may be denoted by x cm. Since x, in the assumed case of a pure tone and simple harmonic vibration, may be taken as the projection, at time t seconds, on the tube axis, of the rotating plane vector

$$x = x_m \epsilon^{j\omega t} \qquad \text{instantaneous cm.} \angle \qquad (22)$$

where ω is the impressed angular velocity of rotation, and x_m is the maximum cyclic vibratory displacement, the vibratory velocity of a transverse layer of air in the tube, also measured along the axis, and in the direction of increasing **l**, is

$$\frac{\partial x}{\partial t} = \dot{x} = j\omega x_m \epsilon^{j\omega t} = j\omega x = \dot{x}_m \epsilon^{j\omega t} \qquad \begin{matrix}\text{inst. kines or} \\ \text{cm./sec.} \angle\end{matrix} \qquad (23)$$

The velocity $\dot{x}$ is also a planevector quantity, capable of being dealt with by the rules of complex arithmetic. Likewise, the vibratory acceleration will be

$$\frac{\partial^2 x}{\partial t^2} = \ddot{x} = j\omega \dot{x}_m \epsilon^{j\omega t} = j\omega \dot{x} = -\omega^2 x_m \epsilon^{j\omega t} = -\omega^2 x \qquad \begin{matrix}\text{inst. kines/sec.} \\ \text{or cm./sec.}^2 \angle\end{matrix} \qquad (24)$$

These equations indicate that in the steady state, the acceleration is in leading quadrature to the velocity, which, in turn, is in leading quadrature to the displacement.

The linear mass, or mass of air normally occupying 1 cm. length of tube will be

$$\mathbf{m} = S\rho \qquad \frac{\text{gm.}}{\text{linear cm.}} \qquad (25)$$

where ρ is the density of the quiescent air in the tube at its actual temperature and its normal pressure intensity p_o dynes per sq. cm.

The fundamental dynamic equation expressing the instantaneous acceleration of a thin layer of air $\partial\mathbf{l}$ cm. long, and having a mass $\mathbf{m}\,\partial\mathbf{l}$ grams is

$$\mathbf{m}\,\ddot{x}\,\partial\mathbf{l} = -\,\partial F \qquad \text{dynes}\angle \qquad (26)$$

where $+\,\partial F$ is the excess of total pressure on the far side of the layer above that on the near side; or

$$\mathbf{m}\,\ddot{x} = S\rho\ddot{x} = -\,\frac{\partial F}{\partial\mathbf{l}} \qquad \frac{\text{dynes}}{\text{linear cm.}}\angle \qquad (27)$$

and[16]

$$\rho\ddot{x} \doteq -\,\frac{\partial p}{\partial\mathbf{l}} \qquad \frac{\text{dynes/sq. cm.}}{\text{linear cm.}}\angle \qquad (28)$$

where p is the excess of the pressure intensity p_o dynes per sq. cm. on the far side of the layer above that on the near side.

It is also a well known acoustic condition [17] that

$$\ddot{x} = v^2\,\frac{\partial^2 x}{\partial\mathbf{l}^2} \qquad \frac{\text{kines}}{\text{sec.}}\angle \qquad (29)$$

where v is the velocity of transmission of sound in the tube, along its axis, in kines.

From (26) and (24)

$$\partial F = -\,\dot{x}\,(j\mathbf{m}\omega)\,\partial\mathbf{l} \qquad \text{rms. dynes}\angle \qquad (30)$$

This expresses the relation between a small difference of alternating pressure ∂F across an elementary length $\partial\mathbf{l}$ of the tube conductor, and the simultaneous vibratory velocity $\dot{x}$ of the element. It corresponds to the well known relation between the difference of alternating

16 Lamb's Hydrodynamics, p. 458, Eq. (3).
17 " " p. 458, Eq. (6).

electric pressure of emf. ∂E, across an elementary length ∂l of a line conductor, and the simultaneous alternating current I in the element

$$\partial E = - I (j\mathfrak{L}\omega)\, \partial l \qquad \text{rms. volts} \angle \quad (31)$$

where $\mathfrak{L}$ is the linear inductance of the conductor in henrys per km., and the linear resistance r of the conductor is ignored. Here ω is the angular velocity of the impressed electric pressure, and l is the length of line in km., measured outwards from the generating end.

Again, from (27) and (29), we have

$$- \frac{\partial F}{\partial \mathbf{l}} = \mathbf{m}v^2 \frac{\partial^2 x}{\partial \mathbf{l}^2} \qquad \frac{\text{dynes}}{\text{linear cm.}} \angle \quad (32)$$

Integrating

$$F = - \mathbf{m}v^2 \frac{\partial x}{\partial \mathbf{l}} + \text{constant} \qquad \text{dynes} \angle \quad (33)$$

The constant of integration vanishes, and may be dropped, because F the pressure deviation from the normal pressure F_o over a cross-section of the tube, vanishes when $\frac{\partial x}{\partial \mathbf{l}} = 0$.

Hence,

$$F\, \partial \mathbf{l} = - \mathbf{m}v^2\, \partial x \qquad \text{cm.-dynes} \angle \quad (34)$$

Differentiating with respect to time:

$$\frac{\partial F}{\partial t}\, \partial \mathbf{l} = \dot{F}\, \partial \mathbf{l} = - \mathbf{m}v^2\, \partial \dot{x} \qquad \frac{\text{cm.-dynes}}{\text{sec.}} \angle \quad (35)$$

or

$$j\omega F\, \partial \mathbf{l} = - \mathbf{m}v^2\, \partial \dot{x} \qquad \frac{\text{cm.-dynes}}{\text{sec.}} \angle \quad (36)$$

and

$$\partial \dot{x} = - \frac{j\omega}{\mathbf{m}v^2} F\, \partial \mathbf{l} \qquad \text{rms. kines} \angle \quad (37)$$

The quantity $\mathbf{m}v^2$ may be replaced by $\mathbf{s}$, the total elastic force resisting compression, expressed in ergs per cm. of displacement over the section; so that the instantaneous differential increase in vibrational velocity over an element of length $\partial \mathbf{l}$ is

$$\partial \dot{x} = - F \left(j \frac{\omega}{\mathbf{s}} \right) \partial \mathbf{l} \qquad \text{rms. kines} \angle \quad (38)$$

The quantity $\mathbf{s}$ is the normal adiabatic elastic force $Sp_o' = S\gamma p_o$, of the medium over the section, γ being the ratio of specific heats, and p_o

the quiescent pressure intensity in the medium. The last equation corresponds to the well known equation of differential increase in alternating current ∂I, over an element of conductor length ∂l; namely

$$\partial \mathrm{I} = -\mathrm{E}(jc\omega)\,\partial l = -\mathrm{E}\left(j\frac{\omega}{s}\right)\partial l \qquad \text{rms. amperes} \angle \qquad (39)$$

where c is the linear capacitance of the conductor in farads per km., and s is its reciprocal. In this case, the linear leakance g of the conductor is ignored.

Since the complete electric conductor equations for (31) and (39), including r and g, are

$$\partial \mathrm{E} = -\mathrm{I}(r + j\mathfrak{L}\omega)\partial l = -\mathrm{I}z\,\partial l \qquad \text{rms. volts} \angle \qquad (40)$$

and

$$\partial \mathrm{I} = -\mathrm{E}\left(g + j\frac{\omega}{s}\right)\partial l = -\mathrm{E}y\,\partial l \qquad \text{rms. amperes} \angle \qquad (41)$$

we may reasonably assume that the corresponding equations for the acoustic conductor, including frictional resistance along the tube and losses due to imperfect elasticity are, from (30) and (38)

$$\partial \mathrm{F} = -\dot{x}(\mathbf{r} + j\mathbf{m}\omega)\,\partial \mathbf{l} = -\dot{x}\,\mathbf{z}\,\partial \mathbf{l} \qquad \text{rms. dynes} \angle \qquad (42)$$

and

$$\partial \dot{x} = -\mathrm{F}\left(\mathbf{g} + j\frac{\omega}{\mathbf{s}}\right)\partial \mathbf{l} = -\mathrm{F}\mathbf{y}\,\partial \mathbf{l} \qquad \text{rms. kines} \angle \qquad (43)$$

In electric conductors, the linear dissipation constants r and g are accepted as constant at any single impressed frequency, whatever values may pertain to E and I; but they are known to change when the impressed frequency is varied over a wide range. Similarly, in acoustic tubes, the linear dissipation constants **r** and **g** are perhaps constant at any single frequency, although they may vary when the frequency is varied. Numerous measurements will have to be collected before the acoustic dissipation constants of a tube can be determined with precision.

The known equations of electric propagation along uniform lines are applicable, following the above theory, to acoustic propagation along uniform tubes, when hydrostatic difference of pressure F is substituted for electric difference of pressure E, vibratory displacement x for alternating quantity q, vibratory velocity $\dot{x}$ for alternating current I, linear mass **m** for linear inductance $\mathfrak{L}$, linear frictional resistance **r** for linear electric resistance r, linear acoustic elasticity loss **g**

for linear electric leakance g, and elastic force $\mathfrak{s}$ for s, the reciprocal of the linear capacitance.

Table I gives a comparison between electric and acoustic quantities for the case of negligible losses in transmission.

Table II gives some comparative data for electric and acoustic conductors, still assumed as having no losses; but freed at the distant end.

Table III gives similar comparative data for the general case of conductors having linear losses of known values.

Surge-Impedance Density: In regard to Table I, it may be noted that the acoustic surge impedance density has no counterpart in the ordinary theory of electric lines.

The value of the surge impedance density, by (49), is $\sqrt{\rho\, p_o'}$ where ρ is the density of quiescent air at 0°C and the standard pressure of 10^6 bars or dynes per sq. cm. The value of $p_o' = \gamma p_o$ may be taken as 1.41×10^6 bars, and $\rho = 1.276 \times 10^{-3}$. Consequently $\mathfrak{z} = \sqrt{17.99 \times 10^2} = 42.4$ mechanic absohms per sq. cm. of tube cross-section, under the standard condition of 1 megabar pressure (10^6 dynes per sq. cm.) and 0°C. If the actual pressure of the quiescent air in the tube is p_o bars, and the actual temperature is t° C., the surge impedance density becomes

$$\mathfrak{z} = \frac{42.4\sqrt{p_o \times 10^{-6}}}{\sqrt{1 + 0.00366\, t}} \quad \frac{\text{mechanic absohms}}{\text{sq. cm.}} \angle \qquad (70)$$

or for values not exceeding 25° C.,

$$\mathfrak{z} = \frac{42.4 \times \sqrt{p_o} \times 10^{-3}}{1 + 0.00183\, t} \quad \frac{\text{mechanic absohms}}{\text{sq. cm.}} \angle \qquad (71)$$

A rigid disk diaphragm of S sq. cm. on each face, vibrating with uniform amplitude over its surface, so as to generate plane waves in the tube indefinitely long, would develop on each face a surge resistance of

$$\mathfrak{z}_o = \frac{42.4\, S \sqrt{p_o} \times 10^{-3}}{1 + 0.00183\, t} \quad \text{mechanic absohms} \angle \qquad (72)$$

In the case of the receiver used in the reported tests, the diameter of the diaphragm inside the clamping ring was 4.96 cm., the corresponding surface area being 19.3 sq. cm. The surge impedance on this surface at 20°C. and 1 megabar pressure, if the diaphragm faced an indefinitely long tube of the same cross-sectional area without acoustic

TABLE I

Properties of Electric and Acoustic Lines with negligible Losses $r = g = 0$, $\mathfrak{r} = \mathfrak{g} = 0$			
Quantity	Electric	Acoustic	
Linear hyperbolic angle	$\alpha = j\omega\sqrt{\mathfrak{c}} = j\omega\sqrt{\frac{\mathfrak{k}}{s}} = j\frac{\omega}{v}$ $= j\frac{2\pi}{\lambda} = j\alpha_2$ hyps $\angle$ / km	$\alpha = j\omega\sqrt{\frac{\mathfrak{m}}{\mathfrak{s}}} = j\frac{\omega}{v} = j\omega\sqrt{\frac{\rho}{p_o'}}$ $= j\frac{2\pi}{\lambda} = j\alpha_2$ hyps $\angle$ / cm	(44)
Hyperbolic angle of conductor	$\theta = \mathrm{L}\alpha = j2\pi\frac{\mathrm{L}}{\lambda} = j\mathrm{L}\alpha_2$ hyps $\angle$	$\theta = \mathrm{L}\alpha = j2\pi\frac{\mathrm{L}}{\lambda} = j\mathrm{L}\alpha_2$ hyps $\angle$	(45)
Wave length	$\lambda = \frac{2\pi}{\alpha_2} = \frac{2\pi}{\alpha}$ km	$\lambda = \frac{2\pi}{\alpha_2} = \frac{2\pi}{\alpha}$ cm	(46)
Velocity of propagation	$v = \frac{\omega}{\alpha_2} = \frac{\omega}{\alpha} = \sqrt{\frac{s}{\mathfrak{k}}} = f\lambda$ km / sec	$v = \frac{\omega}{\alpha_2} = \frac{\omega}{\alpha} = \sqrt{\frac{p_o'}{\rho}} = \sqrt{\frac{\mathfrak{s}}{\mathfrak{m}}} = f\lambda$ kines	(47)
Surge impedance	$\mathfrak{z}_o = \sqrt{\frac{\mathfrak{k}}{c}} = \sqrt{\mathfrak{k}s} = \frac{s}{v}$ ohms	$\mathfrak{z}_o = \sqrt{\mathfrak{m}\mathfrak{s}} = \frac{\mathfrak{s}}{v} = \mathrm{S}\mathfrak{z} = \mathrm{S}\sqrt{p_o'\rho}$ mechanic absohms	(48)
Surge impedance density		$\mathfrak{z} = \frac{\mathfrak{z}_o}{\mathrm{S}} = \frac{\mathfrak{m}v}{\mathrm{S}} = \sqrt{\frac{\rho\mathfrak{s}}{\mathrm{S}}} = \sqrt{p_o'\rho}$ dynes / kine cm²	(49)
Impedance ratio for P and A	$\frac{Z_P}{Z_A} = \frac{\tanh \delta_P}{\tanh \delta_A}$ numeric $\angle$	$\frac{Z_P}{Z_A} = \frac{\tanh \delta_P}{\tanh \delta_A}$ numeric $\angle$	(50)
Power at A	$\mathrm{P}_A = \mathrm{E}_A \mathrm{I}_A$ watts $\angle$	$\mathrm{P}_A = \mathrm{F}_A \dot{x}_A$ abwatts $\angle$ [18]	(51)
Power at A, when $\mathrm{L} = \alpha$	$\mathrm{P}_A = \mathrm{E}_A \mathrm{I}_A = \frac{\mathrm{E}_A^2}{\mathfrak{z}_o} = \mathrm{I}_A^2 \mathfrak{z}_o$ watts $\angle$	$\mathrm{P}_A = \mathrm{F}_A \dot{x}_A = \frac{\mathrm{F}_A^2 v}{\mathfrak{s}} = \dot{x}_A^2 \mathfrak{m} v$ $= \mathrm{S}pkv = \mathrm{S}\dot{x}^2\rho v$ abwatts $\angle$ [18]	(52)

[18] In (51) and (52) either E_A and F_A or I_A and $\dot{x}_A$ must be taken without slope.

TABLE II.

Negligible Losses and distant end free. $r = g = 0$, $\mathbf{r} = \mathbf{g} = 0$, $\delta_B = j\frac{\pi}{2} = j\,1$

Quantity	Electric		Acoustic		
Impedance at Point P	$Z_P = \sqrt{\mathfrak{L}s}\tanh j\,(L_2\alpha_2 + 1)$	ohms ∠	$Z_P = \sqrt{\mathbf{ms}}\tanh (jL_2\alpha_2 + j\,1)$	mech.	(53)
	$= -j\sqrt{\mathfrak{L}s}\cot\left(2\pi\frac{L_2}{\lambda}\right)$	"	$= -j\sqrt{\mathbf{ms}}\cot\left(2\pi\frac{L_2}{\lambda}\right)$	absohms ∠	(54)
	$= -j\sqrt{\mathfrak{L}s}\cot(L_2\alpha_2)$	"	$= -j\sqrt{\mathbf{ms}}\cot(L_2\alpha_2)$		(55)
Impedance ratio for P and A	$\frac{Z_P}{Z_A} = \frac{\cot(L_2\alpha_2)}{\cot(L\alpha_2)}$	numeric	$\frac{Z_P}{Z_A} = \frac{\cot(L_2\alpha_2)}{\cot(L\alpha_2)}$	numeric	(56)
Current / Velocity } " " "	$\frac{I_P}{I_A} = \frac{\sin(L_2\alpha_2)}{\sin(L\alpha_2)}$	"	$\frac{\dot{x}_P}{\dot{x}_A} = \frac{\sin(L_2\alpha_2)}{\sin(L\alpha_2)}$	"	(57)
Voltage / Pressure } " " "	$\frac{E_P}{E_A} = \frac{\cos(L_2\alpha_2)}{\cos(L\alpha_2)}$	"	$\frac{F_P}{F_A} = \frac{\cos(L_2\alpha_2)}{\cos(L\alpha_2)}$	"	(58)
Current / Velocity } at end A	$I_A = \frac{jE_A}{\sqrt{\mathfrak{L}s}\cot(L\alpha_2)}$	amperes ∠	$\dot{x}_A = \frac{j\,F_A}{\sqrt{\mathbf{ms}}\cot(L\alpha_2)}$	kines ∠	(59)
Voltage / Pressure } at end B	$E_B = \frac{E_A}{\cos(L\alpha_2)}$	volts	$F_B = \frac{F_A}{\cos(L\alpha_2)}$	dynes ∠	(60)

TABLE III.

Properties of Electric and Acoustic Lines when Losses are not negligible

Quantity	Electric		Acoustic		
Linear hyperbolic angle	$\alpha = \sqrt{yz} = \sqrt{(r + j\mathfrak{L}\omega)\left(g + j\frac{\omega}{s}\right)}$	$\frac{\text{hyps}}{\text{km}}\angle$	$\alpha = \sqrt{\mathbf{yz}} = \sqrt{(\mathbf{r} + j\mathbf{m}\omega)\left(\mathbf{g} + j\frac{\omega}{\mathbf{s}}\right)}$	$\frac{\text{hyps}}{\text{cm}}\angle$	(61)
	$= \alpha_1 + j\alpha_2$	"	$= \alpha_1 + j\alpha_2$	"	(62)
Hyp. Angle of conductor	$\theta = \mathrm{L}\alpha = \mathrm{L}(\alpha_1 + j\alpha_2) = \theta_1 + j\theta_2$	hyps$\angle$	$\theta = \mathrm{L}\alpha = \mathrm{L}(\alpha_1 + j\alpha_2) = \theta_1 + j\theta_2$	hyp$\angle$	(63)
Wave length	$\lambda = \frac{2\pi}{\alpha_2}$	km	$\lambda = \frac{2\pi}{\alpha_2}$	cm	(64)
Velocity of propagation	$v = \frac{\omega}{\alpha_2}$	$\frac{\text{km}}{\text{sec}}$	$v = \frac{\omega}{\alpha_2}$	$\frac{\text{cm}}{\text{sec}}$	(65)
Surge impedance	$z_o = \sqrt{\frac{z}{y}} = \sqrt{\frac{r + j\mathfrak{L}\omega}{g + j\omega/s}}$	ohms$\angle$	$\mathbf{z}_o = \sqrt{\frac{\mathbf{z}}{\mathbf{y}}} = \sqrt{\frac{\mathbf{r} + j\mathbf{m}\omega}{\mathbf{g} + j\omega/\mathbf{s}}}$	$\frac{\text{dynes}}{\text{kine}}\angle$	(66)
Impedance at point P	$Z_P = z_o \tanh \delta_P$	ohms$\angle$	$Z_P = \mathbf{z}_o \tanh \delta_P$	"	(67)
Impedance ratio for P and A	$\frac{Z_P}{Z_A} = \frac{\tanh \delta_P}{\tanh \delta_A}$	numeric$\angle$	$\frac{Z_P}{Z_A} = \frac{\tanh \delta_P}{\tanh \delta_A}$	numeric$\angle$	(68)
Current / Velocity ratio " "	$\frac{I_P}{I_A} = \frac{\cosh \delta_P}{\cosh \delta_A}$	"	$\frac{\dot{x}_P}{\dot{x}_A} = \frac{\cosh \delta_P}{\cosh \delta_A}$	"	(69)
Voltage / Pressure ratio " "	$\frac{E_P}{E_A} = \frac{\sinh \delta_P}{\sinh \delta_A}$	"	$\frac{F_P}{F_A} = \frac{\sinh \delta_P}{\sinh \delta_A}$	"	(70)

losses, and if the vibration amplitude were uniform over the whole diaphragm, would be $789 \angle 0°$ acoustic absohms. The mass factor of the diaphragm, as measured electrically, with its usual cover opening into the air, was 0.263; so that if this mass factor had been maintained under the geometrical conditions indicated in Figure 3, the acoustic impedance on the outer surface of the diaphragm would be $789 \angle 0° \times 0.263 = 208 \angle 0°$ mechanic absohms. The actual analysis, however, of the results with the tube attached as in Figure 3, made the observed surge impedance on the front surface $187 \angle 5°.2$ mechanic absohms. The discrepancy between the computed and observed value of $\mathbf{z}_o$ may be attributed to three influences (1) acoustic losses in the tube; (2) the sudden change in tube sectional area from 19.3 to 7.55 sq. cm. close to the diaphragm, and (3) a probable change of mass factor m/M under the actual conditions of Figure 3.

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List of Symbols employed.

A	Force factor of a telephone receiver (dynes/absampere $\angle$).
$\alpha = \alpha_1 + j\alpha_2$	Linear hyperbolic angle of an electric or acoustic conductor $\left(\frac{\text{hyps}}{\text{km.}}\angle\right)$ or $\left(\frac{\text{hyps}}{\text{cm.}}\angle\right)$
β°	Slope of force factor, or of a vector quantity (circular degrees).
c	Linear capacitance of electric conductor (farads/km.).
$\gamma = 1.41$	Ratio of specific heats of air or other gas (constant pressure to constant volume) (numeric).
Δ	Apparent damping constant of a telephone receiver (hyps/second).
δ_P, δ_A	Position angles at point P and at sending end A, of electric or acoustic conductor (hyps $\angle$).
E, E_P, E_A, E_B	Electromotive force in a circuit, at a point P on a conductor, at sending end A, and at receiving end B (rms. volts $\angle$).
$\epsilon = 2.718\ldots$	Napierian base (numeric).
F	Vibromotive force on a diaphragm; also at any surface across which vibration occurs (rms. dynes $\angle$).
F_P, F_A, F_B	Vibromotive force over section of acoustic conductor at P, at sending end A or far end B (rms. dynes $\angle$).
f	Impressed frequency (cycles per second).
f_1, f_2	Lower and upper quadrantal frequencies of telephone receiver (cycles/sec.).
$f_\circ$	Frequency of apparent resonance of a receiver (cycles/sec.).
g	Linear leakance of electric conductor (mhos/km.).
$\mathfrak{g}$	Coefficient of linear resilience loss of acoustic conductor (mech. absohms/cm.).
$\theta = \theta_1 + j\theta_2$	Angle subtended by an electric or acoustic conductor (hyps $\angle$).
I	Current strength in a telephone receiver (rms. absamperes $\angle$); also current strength along an electric conductor (rms. amperes $\angle$).
$j = \sqrt{-1}$	
$\kappa = \frac{p}{p'_\circ}$	Compression, or ratio of rms. diminution in volume to quiescent volume (numeric).
L	Length of electric or acoustic conductor (km. or cm.).
L_2	Distance from point P to far end of an electric or acoustic conductor (km. or cm.).
$\mathfrak{L}$	Linear inductance of electric conductor (henrys/km.).
l	Length of an electric conductor (km.).
$\mathfrak{l}$	Length of an acoustic conductor (cm.).
λ	Wavelength on electric or acoustic conductor (km. or cm.).
M	Total mass of a vibrating diaphragm within its clamping circle (gm.).

m, m_d	Equivalent mass of a vibrating diaphragm within its clamping circle (gm.). Equivalent mass of diaphragm in absence of air (gm.).
$\mathbf{m}$	Linear mass of air or other gas in acoustic conductor (gm./cm.).
P	Power at a point on an electric or acoustic conductor (watts or abwatts $\angle$).
p	Ratio of equivalent displacement turns to exciting turns in a telephone receiver (absamperes/cm.).
$p = \frac{F}{S}$	Acoustic alternating pressure (deviation from quiescent hydrostatic pressure) at a cross section of an acoustic conductor (dynes/sq. cm. $\angle$).
p_o	Hydrostatic pressure in quiescent air of acoustic conductor (dynes/sq. cm.).
$p'_o = \gamma p_o$	Adiabatic hydrostatic pressure in quiescent air of tube (dynes/sq. cm.).
$\pi = 3.141....$	(numeric).
q	Quantity of electricity in an a–c. line conductor (rms. coulombs).
R_1	Electric resistance of a telephone receiver as measured with a continuous current (absohms).
R, R_d	Apparent resistance of a telephone receiver to alternating currents when free and damped (absohms).
r	Mechanic resistance of a diaphragm excluding effects of magnetic hysteresis (mech. absohms).
r''	Total mechanic resistance of a diaphragm at apparent resonance (mech. absohms).
r'	Effective mechanic resistance of a diaphragm due to magnetic hysteresis (mech. absohms).
r	Linear resistance of electric conductor (ohms/km.).
$\mathbf{r}$	Linear resistance of acoustic conductor (mech. obsohms/cm.).
ρ	Density of quiescent air or other gas in acoustic conductor (gm./c.c.).
S	Surface area on one side of a diaphragm or cross-sectional area of an acoustic conductor (sq. cm.).
$s = \frac{1}{c}$	Reciprocal of linear capacitance of electric conductor (daraf-km.).
$\mathbf{s} = S\,p'_o$	Resilience force of acoustic conductor over its entire cross-section (dynes).
s_d	Stiffness coefficient of a telephone receiver diaphragm (dynes/cm.).
s'	Apparent change in mechanic stiffness of a diaphragm due to its vibration in a permanent magnetic field (dynes/cm.).
σ	Electric or acoustic load at the B end of a long conductor (ohms $\angle$ or mech. absohms $\angle$).
t	Elapsed time from epoch (seconds).
$t°$	Temperature in gas within acoustic conductor (deg. Cent.).
Λ_o	Oscillatory sharpness of resonance (numeric).
v	Velocity of transmission in electric conductor (km./sec.) and in acoustic conductor (cm./sec.).
X, X_d	Reactance of a telephone receiver free and damped respectively (absohms).

x	Displacement of a diaphragm from its quiescent position or of a thin layer of air in a tube from its quiescent position (rms. cm.).
$\dot{x}$	Vibrational Velocity of a diaphragm or of a thin layer of air in a tube (rms. cm./sec. $\angle$).
$\ddot{x}$	Vibrational acceleration of a diaphragm or of a thin layer of air in a tube (rms. cm./sec.^{2} $\angle$).
x_m, $\dot{x}_m$, $\ddot{x}_m$	Maximum cyclic sizes of vibrational displacement, velocity and acceleration (cm., kines, kines/sec.).
$\underline{x}$, $\underline{\dot{x}}$	Root mean square values of displacement and velocity taken over entire surface of flexible vibrating diaphragm (cm. $\angle$, kines $\angle$).
x_o, $\dot{x}_o$	Root mean square values of displacement and velocity at center of vibrating telephone diaphragm (cm. $\angle$, kines $\angle$).
$Y' = 1/Z'$	Electric motional admittance of telephone receiver (ohms $\angle$).
y	Linear admittance of electric conductor (ohms/km. $\angle$).
$\mathbf{y}$	Linear admittance of acoustic conductor (mech. absohms/cm. $\angle$).
Z, Z_d	Impedance of a telephone receiver, free and damped (absohms $\angle$).
Z_P, Z_A	Impedance at point P and at sending end of electric line (ohms $\angle$) or for acoustic line (mech. absohms $\angle$).
$Z' = Z - Z_d$	Motional impedance of telephone receiver (absohms $\angle$).
Z'_o	Maximum or diametral value of receiver motional impedance (absohms $\angle$).
$\mathbf{z}_1$, $\mathbf{z}_2$, $\mathbf{z}$	Acoustic impedance on sides 1 and 2 of a diaphragm and their vector sum (acoustic absohms $\angle$).
$\mathbf{z}'$	Total mechanic impedance on a diaphragm (mech. absohms $\angle$).
$\mathbf{z}_d$	Internal mechanic impedance of a diaphragm (mech. absohms $\angle$).
$\mathbf{z}_v$	Virtual mechanic impedance of a diaphragm due to its vibration in a permanent magnetic field (mech. absohms $\angle$).
z	Linear impedance of electric conductor (ohms/km.).
$\mathbf{z}$	Linear impedance of acoustic conductor (mech. absohms/cm. $\angle$).
z_o, $\mathbf{z}_o$	Surge impedance of electric and acoustic conductor (ohms $\angle$, mech. absohms $\angle$).
$\mathfrak{z}_1$, $\mathfrak{z}_2$, $\mathfrak{z}$	Acoustic impedance densities on sides 1 and 2 of a diaphragm, and their vector sum (acoustic absohms/sq. cm. $\angle$).
$\omega = 2\pi f$	Impressed angular velocity (radians/sec.).
$\angle$	Sign indicating a planevector quantity or complex number.
rms	Root mean square.
ab or *abs*	Prefix signifying a C.G.S. unit.
a-c	Alternating-current.
d-c	Direct-current.